Tensor forces and relativistic corrections in quarkonium

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Abstract. The influence of tensor forces on hyperfine splitting and decay widths of mesons are investigated within the limits of quasirelativistic potential quark model. As the starting the system of the Rarita-Schwinger equations is used. We use a potential motivated by QCD with a mixed Lorentz structure.

1. Introduction

As well known, the heavy meson spectrum can be successfully obtained starting from a potential model [1-3]. The describing of the fine and hyperfine structure including decay widths is problematic in framework of unified approach. Many authors with different types potentials for quark-antiquark interaction describe only separate properties of mesons. Following papers [4-6] we analyse the Lorentz structure of the potential with reference to $u\bar{u}$ system, charmonium and bottonium hyperfine splitting. It is well known that tensor forces bring about mix wave with different angular momentum [7]. Majority of authors consider that contribution of tensor forces in spectrum of bound states of quarkonia is negligible [8,9]. But, it was shown [3] that D waves give 1 - 5 % contribution for energy spectrum in ground triplet state.

In the present paper we will use model which is based on the quasirelativistic Breit-Fermi Hamiltonian [10] and we will be analyze an influence of tensor forces with reference to two-quark system. Singlet states are described by Schrödinger equation and for triplet states we use the system of Rarita-Schwinger equations [7].

2. Hyperfine splitting

It is widely accepted that the interaction between two quarks (or heavy quark - antiquark) consist of a short range part describing the one-gluon-exchange and a infinitely rising long-range part responsible for confinement of the quarks. This also gives good approximation to the lattice potential. We use

$$V_0 = V_V(r) + V_S(r), \tag{1}$$

where

$$V_V = -\frac{\alpha(r)}{r}, \quad V_S(r) = kr. \tag{2}$$

(The color factor 4/3 has been absorbed in the definition of α). Wilson loop techniques suggest that the confining potential should be taken purely scalar, but relativistic potential calculations which have been published [11-13] show a need for (some) vector confinement. Maintaining $V_V(r)+V_S(r)$ unchanged we do allow a fraction of vector confinement [4,6,13]. Thus

$$V_V(r) = -\frac{\alpha(r)}{r} + \beta_V r, \quad V_S(r) = \beta_S r \quad (\beta_V + \beta_S = k). \tag{3}$$

Then

$$V_0 = (-\frac{\alpha(r)}{r} + \beta_V r) + \beta_S r. \tag{4}$$

The confining potential transforms as the Lorentz scalar and vector potential transforms as the time component of a four-vector potential. As we can see, the choice of Lorentz structure of potential for quark-antquark interaction is important model for study of spin effects [13-17].

We consider the Breit-Fermi Hamiltonian (in case of equal masses $m_1 = m_2 = m$):

$$H = \frac{\overrightarrow{p}^2}{m} + V_0 + H_{LS} + H_{SS} + H_T \tag{5}$$

with spin-orbit term

$$H_{LS} = \frac{1}{2m^2 r}[3\frac{dV_V}{dr} - \frac{dV_S}{dr}](\boldsymbol{L}.\boldsymbol{S}), \tag{6}$$

$\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$ is the total spin of bound state and $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$ is the relative orbital angular momentum of its constituents, the spin-spin term

$$H_{SS} = \frac{2}{3m^2}\Delta V_V(r)\boldsymbol{S}_1.\boldsymbol{S}_2, \tag{7}$$

and the tensor term

$$H_T = \frac{1}{12m^2}[\frac{1}{r}\frac{dV_V}{dr} - \frac{d^2V_V}{dr^2}]S_{12}, \tag{8}$$

where

$$S_{12} = 12[(\boldsymbol{S}_1.\boldsymbol{n})(\boldsymbol{S}_2.\boldsymbol{n}) - (\boldsymbol{S}_1.\boldsymbol{S}_2)/3] = 3(\boldsymbol{\sigma}_1.\boldsymbol{n})(\boldsymbol{\sigma}_2.\boldsymbol{n}) - (\boldsymbol{\sigma}_1.\boldsymbol{\sigma}_2), \tag{9}$$

$$\boldsymbol{n} = \boldsymbol{r}/r, \quad \boldsymbol{S}_i = \boldsymbol{\sigma}_i/2.$$

For bound-state constituents of spin $S_1 = S_2 = 1/2$, the scalar product of their spin, $\boldsymbol{S}_1.\boldsymbol{S}_2$, is given by

$$\boldsymbol{S}_1.\boldsymbol{S}_2 = \begin{cases} -\frac{3}{4} & \text{for spin singlets}, S = 0, \\ +\frac{1}{4} & \text{for spin triplets}, S = 1. \end{cases}$$

Taking picture (4), then (6), (7) and (8) yield

$$H_{LS} = \frac{1}{2m^2}[\frac{3\alpha}{r^3} + \frac{3\beta_V - \beta_S}{r}](\boldsymbol{L}.\boldsymbol{S}), \tag{10}$$

$$H_{SS} = \frac{2}{3m^2}[4\pi\alpha\delta(\boldsymbol{r}) + \frac{2\beta_V}{r}](\boldsymbol{S}_1.\boldsymbol{S}_2), \tag{11}$$

$$H_T = \frac{1}{12m^2}(\frac{3\alpha}{r^3} + \frac{\beta_V}{r})S_{12}. \tag{12}$$

Two-quark system may exist in the spin singlet and the triplet states (Table 1).

If $S = 0$, then necessarily $L = J$ and eigenfunction is of the form $(f(r)/r)\mathcal{Y}^M_{J0J}$, where

$$\mathcal{Y}^M_{JLS} = \sum_{m\mu}\langle LSm\mu|JM\rangle\langle Y_{Lm}(\Omega)|S\mu\rangle. \tag{13}$$

Since

$$\boldsymbol{S}|00 >= 0,$$

$$S_{12} = 2[3(\boldsymbol{S}.\boldsymbol{n})^2 - \boldsymbol{S}^2] \tag{14}$$

we get

$$S_{12}\mathcal{Y}^M_{JL0} \equiv S_{12}Y_{Jm}(\Omega)|00>=0.$$

Consequently, $f(r)$ satisfies radial equation

$$\frac{d^2f}{dr^2}+[k^2-\frac{L(L+1)}{r^2}-^1U_c]f=0, \tag{15}$$

where ${}^1U_c = m^1V_c;^1V_c = V_V(r)+V_S(r)-(3/4)V_{SS}$ and $k^2=mE$. If $S=1$ and $P=(-1)^{J+1}$, the only possible values of L are $J+1$ (unless $J=0$ in which case there is just the one value $L=1$).

The wave function for ground triplet state of $q\bar{q}$ system with negative parity ($P=-1$) is a mixture of state ${}^3S_1$ and ${}^3D_1$ and may be put in the form

$$\psi=\psi_S+\psi_D \equiv \frac{1}{r}u(r)\mathcal{Y}^1_{101}+\frac{1}{r}w(r)\mathcal{Y}^1_{121}. \tag{16}$$

The input D-waves to the wave function of vector meson makes part of percent. But the input D-wave to the spectrum of energy makes some percent. It is explained by including of the interference member to the energy spectra. Knowing the wave function it is possible to calculate input of every component to the energy level. We have

$$E=\langle\psi_S+\psi_D|H|\psi_S+\psi_D\rangle.$$

Then the energy value can be split into components

$$E=E_S+E_D+E_{SD},$$

where E_S, E_D and represent inputs of S, D-wave and of the interference wave, respectively.

Now, for $S=1$ state we have the wave function (16) we know that,

$$S_{12}\mathcal{Y}^1_{010}=\sqrt{8}\mathcal{Y}^1_{121}$$

and

$$S_{12}\mathcal{Y}^1_{121}=\sqrt{8}\mathcal{Y}^1_{101}-2\mathcal{Y}^1_{121}.$$

Futher

$$\boldsymbol{L}^2\mathcal{Y}^1_{101}=0, \quad \boldsymbol{L}^2\mathcal{Y}^1_{121}=6\mathcal{Y}^1_{121}.$$

Since

$$H = -\frac{1}{m}\frac{d^2}{dr^2} + \frac{\boldsymbol{L}^2}{mr^2} +^3 V_c + V_{LS}\boldsymbol{L}.\boldsymbol{S} + V_T S_{12},$$

and $\boldsymbol{S}_1.\boldsymbol{S}_2 = 1/4, \boldsymbol{L}.\boldsymbol{S} = -3, S_{12} = -2$ for $S = 1$. Then the equation $(H - E)\psi = 0$ is equivalent to the Rarita-Schwinger system

$$[-\frac{1}{m}\frac{d^2}{dr^2} - E +^3 V_c]u + \sqrt{8}V_T w = 0,$$

$$[-\frac{1}{m}\frac{d^2}{dr^2} - E + \frac{6}{mr^2} +^3 V_c - 2V_T - 3V_{LS}]w + \sqrt{8}V_T u = 0, \tag{17}$$

where

$$^3V_c = V_V + V_S + \frac{1}{4}V_{SS}.$$

The system (17) we rewrite in the matrix form

$$\widehat{h}\begin{pmatrix} u \\ w \end{pmatrix} = E\begin{pmatrix} u \\ w \end{pmatrix},$$

with

$$\widehat{h} = \widehat{h}_0 + \widehat{W}$$

and

$$\widehat{h}_0 = \begin{pmatrix} -\frac{1}{m}\frac{d^2}{dr^2} - \frac{\alpha}{r} + kr + \frac{1}{m^2 r}\frac{\beta_V}{3} & \frac{1}{m^2 r}\frac{\sqrt{2}\beta_V}{6} \\ \frac{1}{m^2 r}\frac{\sqrt{2}\beta_V}{6} & -\frac{1}{m}\frac{d^2}{dr^2} + \frac{6}{mr^2} - \frac{\alpha}{r} + kr + \frac{1}{m^2 r}(-\frac{13}{3}\beta_V + \frac{3}{2}\beta_S) \end{pmatrix}, \tag{18}$$

$$\widehat{W} = \begin{pmatrix} \frac{1}{4}\Delta V_{SS} & \frac{\sqrt{8}}{12}\Delta V_T \\ \frac{\sqrt{8}}{12}\Delta V_T & \frac{1}{4}\Delta V_S S - \frac{2}{12}\Delta V_T - 3\Delta V_{LS} \end{pmatrix}, \tag{19}$$

where

$$\Delta V_{SS} = \frac{8\pi\alpha}{3m^2}\delta(\vec{r}), \quad \Delta V_{LS} = \frac{3\alpha}{2m^2 r^3}, \quad \Delta V_T = \frac{3\alpha}{m^2 r^3}.$$

Then matrix elements of correction terms are

$$\begin{aligned} \Delta E_{mn} &= \int u_m(\frac{2\pi\alpha}{3m^2}\delta(\boldsymbol{r}))u_n dr + \int w_m(\frac{\alpha\sqrt{2}}{2m^2 r^3})u_n dr \\ &+ \int u_m(\frac{\alpha\sqrt{2}}{2m^2 r^3})w_n dr + \int w_m(\frac{2\pi\alpha}{3m^2}\delta(\boldsymbol{r}) - \frac{5\alpha}{m^2 r^3})w_n dr. \end{aligned} \tag{20}$$

We put $\alpha = \frac{4}{3}\alpha_s$. The QCD running constant α_s is determined according to the formula

$$\alpha_s(q^2) = 12\pi/[(33 - 2n_j)ln(q^2/\wedge^2)],$$

where $\wedge = \wedge_{QCD} = 140MeV$; $n_j = 3$ for light and mixed mesons; $n_j = 4$ for $c\bar{c}$- and $b\bar{b}$-quarkonium. For Cornell potential hyperfine splitting of mesons may be described with a rather accuracy if $q = 2\mu$ (μ - reduced mass) is chosen [18].

In fact ΔE_{mn} have singularities ($\delta(\boldsymbol{r})$) and r^{-3}. Therefore one has to introduce smeared δ function and cut of for r^{-3} to weaken singularities. Our calculation were made with tensor forces and without tensor forces. We have used the following parameters of potentials: $\beta_V = 0.001GeV^2, \beta_S = 0.179GeV^2$ for $u\bar{u}$-systems; $\beta_V = 0.04GeV^2, \beta_S = 0.14GeV^2$ for charmonium and bottonium, $\alpha_s(c\bar{c}) = 0.38, \alpha_s(b\bar{b}) = 0.24, \alpha_s(u\bar{u}) = 0.54$. The quark masses is: $m_c = 1.4GeV, m_b = 4.7GeV$ and $m_u = 0.33GeV$. In the works [13,16] it was pointed out that the best agreement is obtained when $\beta = 0.18GeV^2$. During the description of $q\bar{q}$-system we were changing the parameter β_V (when $\beta_V + \beta_S = 0.18GeV^2$) to achieve the agreement with experimental mass splitting of 1^{--}-states. Tables 2 - 4 list calculated results. We also display experimental magnitudes [19].

Tables 2 - 4 present the mass spectra of pseudoscalar and vector mesons. They also give the size of D-wave admixture in the wave function of vector meson and the input of interferential member E_{SD}.

We also have calculated root-mean-square radii and results are provided in Table 5.

Squared mean value radius 0.7 fm meet the requirements of quark-antiquark pair creation ($u\bar{u}$) (string break). It is necessary modify the potential model taking into account the opening of a new channel for those conditions where the given value is overlapped.

3. Leptonic decay of heavy quarkonia

For the leptonic decay widths of two-quark system we shall consider decay of ^3S (vector) states into e^+e^- pairs. The leptonic decay width of system $Mq\bar{q} \to e^+e^-$ is calculated from the Van Royen Weisskopf formula [22]

$$\tilde{\Gamma}(^3S_1 \to e^+e^-) = \frac{4\alpha_{em}^2 Q^2}{M_{q\bar{q}}}|R(0)|^2. \tag{21}$$

Where $M_{q\bar{q}}$ is mass of vector meson, Q is quark charge, α_{em} is the fine structure constant and $R(0)$ is the radial wave function at the origin.

The formula (21) is based on the notion that constituent quark-antiquark pair annihilates into a single virtual foton, which subsequently gives rise to a leptonic pair.

The decay widths of the vector $b\bar{b}$ and $c\bar{c}$ quarkonia into charged lepton [23]

$$\Gamma(^3S_1 \to e^+e^-) = \tilde{\Gamma}(1 - \frac{16\alpha_s(m_q^2)}{3\pi}). \tag{22}$$

As Eichten and Quigg have pointed out [24] the QCD correction reduces the magnitude of Γ significantly, however the amount of reduction is somewhat uncertain . For vector mesons containing light quarks this formula leads to paradoxes [25].

In paper [2] Motyka and Zalewski have got formula

$$\Gamma_{V \to e^+e^-} = F(q)\frac{32\alpha_s}{9M_V^2}|R(0)|^2, \tag{23}$$

with $F(c) = 4.73.10^{-5}$ for charmonium and $F(b) = 2.33.10^{-5}$ for bottonium. We have calculated decay widths by Van Royen-Weisskopf (21) and by (23). Table 6 lists results of calculation. The calculations of widths were done with tensor forces and without tensor forces. Value of the widths which were calculated by formula (23) are given in parentheses.

4. Conclusion

The analysis of our results presented in Table 2 - 4 shows that percentage differences between the theoretical calculations of mass spectrum for heavy quarkonia and the experimental results are 1 - 4 %. Therefore we expect relativistic correction in the range 1 - 16 % for charmonium, and up to 1 % for bottonium spectrum. Also we have been able to describe hyperfine splitting of $c\bar{c}$- and $b\bar{b}$-quarkonium. For describing mass spectrum of light mesons it is necessary to use relativistic potential model. We have calculated hyperfine splitting $u\bar{u}$ system, too.

The leptonic decay widths suggest that for charmonium theoretical widths, which were calculated by Van Royen-Weisskopf formula are systematically higher than experimental data. But for bottonium we have obtained lower values. For J/ψ-meson better decay widths are obtained by means of formula (23), which takes into account QCD correction.

Here the influence of D-waves ranges from 4 % for ground state up to 25 % for the second excited state, but for values calculated by Van Royen-Weisskopf formula it is from 8 % up to 50 %. For Υ-meson it is opposite. More exact is calculation done by (21), but QCD correction reduced the results more apparent. Besides, D-waves contribute less than for charmonium: from 4 % for ground state up to 11 % for second excited state.

The results obtained show that contribution of the D-waves is impossible to neglect for considered leptonic decay width of quarkonia.

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Table 1: **Conditions of two fermion system**

	Singlet condition ($S=0$)		Triplet condition ($S=1$)	
J/P	+	-	+	-
0	- - - -	$^1S_0$	$^3P_0$	- - - -
1	$^1P_1$	- - - -	$^3P_1$	$^3S_1 +^3 D_1$
2	- - - -	$^1D_2$	$^3P_2 +^3 F_2$	$^3D_2$

Table 2: **Hyperfine splitting for the charmonium**

State	S-wave E_{theor}, MeV	SD-waves E_{theor}, MeV	[20] E_{theor}, MeV	[19] E_{exp}, MeV	E_{SD}, %	P_D, %
1^1S_0	2980			2980		
1^3S_1	3153	3097		3097	16	0.05
$1^3S_1 - 1^1S_0$	173	117	110	117		
2^1S_0	3642			3590		
2^3S_1	3759	3734		3685	3	0.8
$2^3S_1 - 2^1S_0$	117	92	67	95		
3^1S_0	4107			-		
3^3S_1	4208	4192		4040	1	1.3
$3^3S_1 - 3^1S_0$	101	85		-		

Table 3: **Hyperfine splitting for the bottonium**

State	S-wave E_{theor}, MeV	SD-waves E_{theor}, MeV	[20] E_{theor}, MeV	[19] E_{exp}, MeV	E_{SD}, %	P_D, %
1^1S_0	9415			-		
1^3S_1	9462	9460		9460	2.1	0.004
$1^3S_1 - 1^1S_0$	47	45	46	-		
2^1S_0	9883			-		
2^3S_1	9911	9911		10023	0.2	0.04
$2^3S_1 - 2^1S_0$	28	28	26	-		
3^1S_0	10201			-		
3^3S_1	10224	10224		10355	0.1	0.1
$3^3S_1 - 3^1S_0$	23	23		-		

Table 4: **Hyperfine splitting for** $(u\vec{u})$**-systems**

State	S-wave E_{theor}, MeV	SD-waves E_{theor}, MeV	[20] E_{theor}, MeV	[19] E_{exp}, MeV	E_{SD}, %	P_D, %
1^1S_0	140			140		
1^3S_1	674	640		770	4	0.02
$1^3S_1 - 1^1S_0$	534	500	923	630		
2^1S_0	1134			1300		
2^3S_1	1564	1543		1450	1	0.04
$2^3S_1 - 2^1S_0$	430	409	411	150		

Table 5: **Squared mean value radius**

nL	[21] $\langle r_c^2\rangle^{1/2}$, fm	Our results $\langle r_c^2\rangle^{1/2}$,fm	[21] $\langle r_b^2\rangle^{1/2}$, fm	Our results $\langle r_b^2\rangle^{1/2}$, fm
1 S	0.43	0.433	0.24	0.256
2 S	0.85	0.847	0.51	0.552
3 S	1.18	1.182	0.73	0.768

Table 6: **The leptonic decay widths of heavy mesons**

State	SD-waves $\Gamma_{theor.}$,keV	S-wave $\Gamma_{theor.}$,keV	[2] $\Gamma_{theor.}$,keV	[26] $\Gamma_{theor.}$,keV	[27] $\Gamma_{theor.}$,keV	[19] $\Gamma_{exp.}$,keV
$J/\psi 1S$	7.8 (5.41)	8.2 (5.63)	4.5	4.24	8.0	5.26 ± 0.37
$\psi' 2S$	3.7 (2.59)	4.0(2.79)	1.9	1.81	3.7	2.12 ± 0.18
$\psi'' 3S$	2.6 (1.82)	2.9 (2.01)	- - -	1.22	- - -	0.75 ± 0.15
$\Upsilon 1S$	1.14 (0.96)	1.20 (1.01)	1.36	0.85	1.7	1.32 ± 0.04
$\Upsilon' 2S$	0.58 (0.49)	0.63 (0.53)	0.59	0.38	0.8	0.52 ± 0.03
$\Upsilon'' 3S$	0.44 (0.37)	0.49 (0.42)	0.40	0.27	0.6	0.48 ± 0.08